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The free electron laser with inverse taper

W.B. Colson*, R.D. McGinnis

Physics Department, Naval Postgraduate School, 833 Dyers Road, Monterey, CA 93943, USA

Abstract

The undulator design determines the physics of the free electron laser interaction. In the tapered undulator, the resonant electron energy decreases along the undulator length in order to enhance gain and efficiency in strong optical fields. With inverse taper, the resonant electron energy increases along the undulator length. Surprisingly, gain and efficiency are also increased in strong optical fields. In addition, the resultant electron beam energy spread is decreased compared to both the tapered and untapered undulator. A reduction of the final electron beam energy spread is useful when recirculating the beam current as in the Jefferson Lab FEL.

1. Introduction

In the free electron laser (FEL), a relativistic beam of electrons passes through a magnetic undulator to amplify co-propagating laser light [1]. During the interaction, about half the electrons in the beam lose energy to the laser light while the others take energy away from the laser. This process induces an energy spread in the electron beam that eventually leads to saturation and gain reduction in strong optical fields. In an FEL that recirculates the electron beam, as does the Thomas Jefferson National Accelerator Facility (TJNAF), the induced energy spread can limit the system performance [2]. When an electron beam with a broad energy spectrum passes through bending magnets, the resulting angular spread makes the beam transport difficult.

The design of the undulator can be modified to alter the laser/electron interaction and the final electron beam distribution. Several such designs have been explored theoretically and experimentally for varied purposes. The tapered undulator [3] decreases the undulator wavelength λ_0, or the undulator field strength, along the undulator length in order to maintain resonance with electrons that lose energy to the laser light. The tapered undulator increases gain and efficiency in strong optical fields and extends the usual saturation limit to stronger optical fields. At first sight, it may seem that there is no reason to consider an inverse taper, increasing the undulator wavelength or undulator field strength, but it appears there are several possible advantages. The inverse taper has been the subject of experimental [4–6] and theoretical research [7], but the goals here are somewhat different.

The resonance between the undulator, laser light of wavelength λ, and relativistic electrons with z velocity $c\beta_z$ along the undulator axis, is determined by the electron phase velocity $v = \dot{\zeta}$, where ζ is the electron phase that follows the microscopic evolution of electron bunching and $(\dot{\cdot\cdot}) = \mathrm{d}(\cdot\cdot)/\mathrm{d}\tau$, where τ is the dimensionless time, $\tau = ct/L$, so that

* Corresponding author. Tel.: + 1-408-656-2765; fax: + 1-408-656-2834.

E-mail address: colson@physics.nps.navy.mil (W.B. Colson).

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$\tau = 0 \rightarrow 1$ along the undulator length L and c is the speed of light [1]. At resonance, $\nu = 0$, we have $\lambda = \lambda_0(1 - \beta_z)$ It is only near resonance that the coupling between the laser and electron beams is significant, but the detailed gain spectrum near resonance can be complicated and depends on the optical field strength.

Tapering the undulator properties can be achieved by increasing or decreasing the undulator wavelength λ_0, along z, increasing or decreasing the undulator field strength B along z, or both. Each method of tapering the undulator is conceptually equivalent and causes an electron phase acceleration, $\nu = \delta$ in time τ as electrons travel along the undulator length $L = N\lambda_0$ with N periods. We assume here that the tapering is such that $\delta = L^2 \mathrm{d}k_0(z)/\mathrm{d}z$ is constant along the undulator where $k_0 = 2\pi/\lambda_0$. For an undulator with $N = 25$ periods and a 12% wavelength decrease along its length, the value of the phase acceleration is $\delta = 6\pi$ A 12% wavelength increase gives $\delta = -6\pi$, and a conventional periodic undulator has $\delta = 0$.

The electron dynamics in a tapered undulator are described by the pendulum equation with an additional torque δ,

$$\ddot{\zeta} = \dot{\nu} = \delta + a\cos(\zeta) \tag{1}$$

where a is the dimensionless optical field strength [1]. Together with the self-consistent wave equation, the theory is valid in weak and strong optical fields with either high or low gain.

2. The periodic undulator

In the conventional periodic undulator with weak initial optical field $a_0 \leq \pi$, the gain spectrum $G \approx (\nu_0)$ is well-known to be antisymmetric about resonance with peak value of $G \approx 0.13j \approx 26\%$ at $\nu_0 = 2.6$ where ν_0 is the initial electron phase velocity and $j = 2$ is taken as a typical dimensionless electron beam current density [1]. The gain spectrum width is $\Delta\nu_0 \approx \pi$, and the gain is zero at resonance $\nu_0 = 0$. In the FEL oscillator, while the optical fields are small and still evolving from spontaneous emission, the optical wavelength follows the peak gain at $\nu_0 \approx 2.6$.

As the optical fields grow to saturation $a \approx 4\pi^2 \approx 40$, the gain spectrum $G(\nu_0)$ changes its shape. While it remains antisymmetric about $\nu_0 = 0$, the peak available gain reduces to only $G = 1\%$ at $\nu_0 \approx 10$ and the gain spectrum broadens. At saturation, the FEL oscillator wavelength must drift from $\nu_0 \approx 2.6$ to 10, corresponding to about 5% for $N = 25$.

In the electron pendulum phase space determined by Eq. (1), saturation in strong fields occurs when the electrons become trapped in closed orbits. The pendulum separatrix height, $2\sqrt{a}$, becomes greater than the phase velocity for peak gain in weak fields, $\nu_0 = 2.6$, then an increasingly larger fraction of the beam becomes trapped in the closed orbit region of phase space. At saturation, the FEL interaction induces a spread in phase velocities almost equal to the height of the separatrix, $\Delta\nu \approx 4\sqrt{a} \approx 8\pi$, or about 10% for $N = 25$. This spread can limit the recirculation of the electron beam.

3. The tapered undulator

In the tapered undulator in weak fields, the gain spectrum $G(\nu_0)$ is no longer antisymmetric about resonance. The peak gain is reduced to only $G \approx 13\%$ for $j = 2$ or about half the value of the periodic undulator, and peaks at $\nu_0 \approx 2.6 - \delta/2 \approx -7$ with a taper rate of $\delta = 6\pi$. The gain spectrum width is still $\Delta\nu_0 \approx \pi$.

While weak-field gain is reduced in the tapered undulator, gain in strong fields is increased. The gain spectrum distorts as the periodic undulator in that peak gain decreases and moves to larger values of ν_0 and the gain spectrum broadens. In fact, the peak gain in strong fields occurs at $\nu_0 \approx 0$ and is improved to $G \approx 2\%$ instead of only 1% for the periodic undulator. During the evolution of the optical fields to saturation, the FEL oscillator wavelength must drift from $\nu_0 \approx -7$ to 0, giving the same 5% shift as in the periodic undulator.

In the tapered undulator, the phase space paths are modified by pendulum torque δ. When the taper rate is too large, so that $\delta > a$, the electron phase space has no closed orbits. In sufficiently strong fields, $a > \delta$, some electrons can be "trapped" in the closed orbits of the pendulum phase space centered near the relative phase $\zeta \approx \cos^{-1}(-\delta/a)$. In stronger fields $a \gg \delta$, a large raction of the electrons can be trapped near resonance around $\zeta \approx \pi$ and continue to lose energy to the optical field. The separatrix surrounding the closed-orbit paths in phase space is given by $v_s^2 = 2\delta(\zeta_s - \zeta_0) + 2a(\sin(\zeta_s) - \sin(\zeta_0))$ where $\zeta_0 = 2\pi - \cos^{-1}(-\delta/)$. For $\delta > a$, the $\cos^{-1}$ has no solution, and there is no separatrix. For $\delta = 0$, we recover the expression for the separatrix in a periodic undulator.

Electrons starting near the phase $\zeta \approx 0$ are accelerated by both the torque δ and the strong optical field a, while the electrons starting near $\zeta \approx \pi$ see the torques δ and a roughly cancel leaving them trapped in closed orbits. Those that are accelerated away from resonance eventually contribute less to the interaction. In this view, tapering is effective because electrons near the phase for gain $\zeta \approx \pi$ are trapped, while electrons near the phase for absorption $\zeta \approx 0$ are taken away from resonance and eventually stop interacting; the imbalance leads to net gain.

The electrons in phase space are spread between the trapped electrons near resonance $v \approx 0$ and the untrapped electrons at $v \approx a$. In a strong field of $a = 40$ and with taper rate $\delta = 6\pi$, the induced spread is 13% with about half of the electrons at each extreme, trapped and untrapped. The tapered undulator does not appear to be desirable for recirculating the electron beam to recover energy.

4. The inverse-taper undulator

In the inverse-taper undulator, $\delta < 0$. Fig. 1 shows the gain spectrum for an FEL with inverse taper rate $\delta = -6\pi$ over $N = 24$ periods with a current density of $j = 2$. These parameters are descriptive of the TJNAF FEL with recirculation of the electron beam. In weak fields ($a \le \pi$), the gain spectrum peak is shifted above resonance to $v_0 \approx 2.6 - \delta/2 \approx 12$ with peak value $G \approx 12\%$ comparable to the tapered case. The gain spectrum width is again $\Delta v_0 \approx \pi$. Both taper and inverse taper have the effect of shifting the gain spectrum by $v_0 \approx 2.6 - \delta/2$ and reducing the peak gain by 50% with $\delta = \pm 6\pi$.

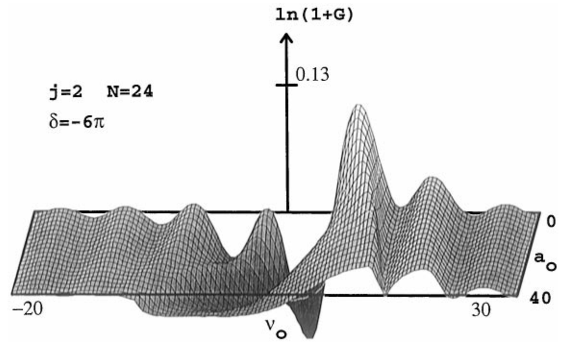

Fig. 1. FEL gain spectrum $G(v_0, a_0)$ with inverse rate $\delta = -6\pi$.

In stronger fields, the gain spectrum distorts, but in a manner differing from both the periodic and tapered undulators. The peak in the gain spectrum stays at nearly the same value of v_0 in both weak and strong fields. In the FEL oscillator with inverse taper, the wavelength does not shift as the FEL saturates. The peak available gain decreases, but only to $G \approx 3\%$, so there is a significant advantage over the tapered undulator even in strong fields. This is unanticipated, since the tapered undulator has long been thought to provide the best extension of the FEL to strong fields.

The phase space picture points to another advantage of the inverse taper design. The separatrix surrounding the closed orbits is still given by the expression above. Fig. 2 shows the separatrix in the pendulum phase space plot of (ζ, v) for 5000 sample electrons which have evolved along an undulator with inverse taper rate $\delta = -6\pi$ over $N = 24$ periods with current density $j = 2$. The electrons started at $v_0 = 15$ for peak gain in strong field $a = 40$ with a small Gaussian energy spread of standard deviation $\sigma_G = 1$. All the electrons start on open phase space paths and cannot get into the closed-orbit region outlined by the separatrix. Following the open-orbit paths, it is clear that the electrons must bunch to avoid the closed-orbit region. The bunch continues to follow the paths down to final phase-space sites around $v \approx -20$ where they have lost considerable energy. The induced spread in the final electron distribution is

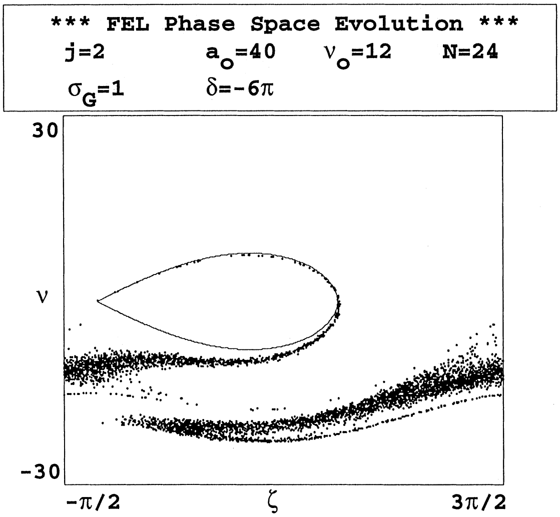

Fig. 2. FEL phase space evolution with inverse rate $\delta = -6\pi$.

$\Delta\nu \approx 15$ corresponding to only a 4% energy spread with a small tail extending to resonance. This is significantly reduced compared to the tapered undulator spread of 13% and the periodic undulator's 10% spread.

5. Conclusion

In summary, the inverse tapered undulator is no more difficult to construct than the tapered undulator, but has advantages over both the periodic and tapered undulators. The advantages are (1) a better gain in strong field, 3% versus 2% and 1%, (2) a small wavelength shift for the FEL oscillator at saturation, and (3) a smaller induced energy spread 4%, instead of 13% and 10%. A more complete description of the inverse-tapered FEL will discuss short pulses.

Acknowledgements

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